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Sound Propagation Through the Elliptical SVU

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A conceptual model for window manufacturing which is capable of ventilation, regulating sunlight, and reducing a traffic and environment noise has been presented in previous studies. This window combines two basic components: a soundproofing ventilation unit (SVU) and a lighting unit. Due to the fact that the ventilation unit must have a large volume to attenuate low-frequency noise, many resonance of higher-order mode wave will be generated inside the unit. To minimize the higher-order mode in order to have a great soundproofing effect, an elliptical cavity is take into consideration in this paper. At first, by using the Mathieu function, the general expression of the output pressure to the given input uniform velocity is obtained for a whole ventilation unit. Next, based on the calculation results, the cause and mechanism of resonance frequencies inside an element are discussed in detail.

Keywords: soundproofing ventilation unit, higher-order mode wave, sound propagation

1. INTRODUCTION

Sealing up type's doors and windows are widely used in the current houses to intercept an inside outside. Needless to say, some home equipment such as air conditioners are necessary to keep a comfortable indoor's temperature while such doors and windows are closed. In previous study, we have been presented a conceptual model for window manufacturing which is capable of ventilating, regulating sunlight and reducing traffic noise for the developing tropical countries [1]. The ventilation unit is constructed using rectangular cavity with input and output openings at both ends. Due to the fact that the ventilation unit must have a large volume to attenuate low-frequency noise, many resonance of higher-order mode wave will be generated inside the unit. To minimize the higher-order mode in order to have a great soundproofing effect, an elliptical cavity is take into consideration in this paper. The ventilation unit is model as a piston-driven elliptical rigid tube with no losses. By using the Mathieu function the general expression of the output pressure to the given input uniform velocity is obtained for a whole ventilation unit. Based on the calculation results, the cause and mechanism of resonance frequencies inside an element are discussed in detail.

2. METHOD OF ANALYSIS

A new type of window combines two basic components: ventilation and lighting(SVU) as shown in **Fig. 1**. The lighting unit can be constructed using one or two glass layers which are mounted between two ventilation components with input and output openings. The ventilation unit can be constructed using wood or metals, it also serves as an import function in reducing noise, which we must design using acoustics technology. Note that sound propagating through this element is a combination of two kinds of waves: standing and higher-order mode waves. Thus, we need to choose a shape that minimizes the higher-order mode wave component as much as possible.

Here, we present the theoretical calculation of the sound pressure inside the elliptical unit including the effects of higher-order mode wave for a simple case where no acoustic material is used. Model of the unit is shown in **Fig. 2**. A section area S_w and length L of elliptical cavity that has an input and output at both side, the sectional area of them are S_0 and S_L, respectively.

The complete solution of wave equation when expressed in elliptical coordinates is [2].

$$\phi = (A_0 \exp(\mu z) + B_0 \exp(-\mu z)) \left(\sum_{m=0}^{\infty} C_m Ce_m(\xi,s)\, Se_{m+1}(\xi,s)\, ce_m(\eta,s)\, se_{m+1}(\eta,s) + \sum_{m=0}^{\infty} S_{m+1} Se_{m+1}(\xi,s) se_{m+1}(\eta,s) \right) \quad (1)$$

where $ce_m(\eta,s)$ $se_{m+1}(\eta,s)$ and $Ce_m(\xi,s)$ $Se_{m+1}(\xi,s)$ are the Mathieu function and modified Mathieu function of mth-order, respectively. Other symbols are constants.

Let $V_x = -\partial\phi / \partial x$, $V_y = -\partial\phi / \partial y$ and $V_z = -\partial\phi / \partial z$ be the velocity components in the x, y and z directions, respectively.

The boundary conditions are

{1} at z=0 $V_z = -\partial\phi / \partial z = V_0 F_0(\xi,\eta)$ (2)

{2} at z=L $V_z = -\partial\phi / \partial z = V_L F_L(\xi,\eta)$ (3)

{3} at $\xi = \xi_w$ $V_\xi = -\partial\phi / \partial\xi = 0$ (4)

where

$$F_0(\xi,\eta) = \begin{cases} 1 & (\xi_{01} \le \xi \le \xi_{02}), (\eta_{01} \le \eta \le \eta_{02}) \\ 0 & elsewhere \end{cases} \quad (5)$$

$$F_L(\xi,\eta) = \begin{cases} 1 & (\xi_{L1} \le \xi \le \xi_{L2}), (\eta_{L1} \le \eta \le \eta_{L2}) \\ 0 & elsewhere \end{cases} \quad (6)$$

Based on the boundary conditions as given above, we will find the velocity potential ϕ.

At first, from boundary condition {2}

$$\frac{\partial}{\partial z}(A_0 \exp(\mu z) + B_0 \exp(-\mu z)) \Big|_{z=L} = 0$$

$$\therefore \quad B_0 = A_0 \exp(2\mu L) \quad (7)$$

Substituting Eq. (7) into Eq. (1), we have

$$\phi = f(\mu,z) \sum_{m=0}^{\infty} \left(C_m Ce_m(\xi,s) ce_m(\eta,s) + S_{m+1} Se_{m+1}(\xi,s) se_{m+1}(\eta,s) \right) \quad (8)$$

Where

$$f(\mu,z) = \exp(\mu z) + \exp(2\mu L)\exp(-\mu z) \quad (9)$$

From boundary condition {3}

$$\frac{\partial}{\partial \xi}\left(\sum_{m=0}^{\infty} \left(C_m Ce_m(\xi,s) ce_m(\eta,s) + S_{m+1} Se_{m+1}(\xi,s) se_{m+1}(\eta,s) \right) \right)\Big|_{\xi=\xi_w} = 0 \quad (10)$$

Equation (10) is satisfied by requiring that s have those values that make

$$Ce_m'(\xi,s) = \frac{\partial}{\partial \xi} Ce_m(\xi,s) \Big|_{\xi=\xi_w} = 0 \quad (11)$$

$$Se'_{m+1}(\xi,s)=\frac{\partial}{\partial\xi}Se_{m+1}(\xi,s)\bigg|_{\xi=\xi_w}=0 \qquad (12)$$

Let the positive parametric roots of Eq. (11) and Eq. (12) are designated as $s_{m,i}$ and $\bar{s}_{m,i}$, respectively. Then Eq. (8) becomes

$$\phi=\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} f(\mu_{m,i},z)$$
$$C_{m,i}Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$
$$+\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} f(\bar{\mu}_{m+1,i},z)$$
$$S_{m+1,i}Se_{m+1}(\xi,\bar{s}_{m+1,i})se_{m+1}(\eta,\bar{s}_{m+1,i}) \qquad (13)$$

From boundary condition {1}, we have

$$-V_iF_i(\xi,\eta)-V_0F_0(\xi,\eta)$$
$$=\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} f'(\mu_{m,i},0)$$
$$C_{m,i}Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$
$$+\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} f'(\bar{\mu}_{m+1,i},0)$$
$$S_{m+1,i}Se_{m+1}(\xi,\bar{s}_{m+1,i})se_{m+1}(\eta,\bar{s}_{m+1,i}) \qquad (14)$$

By multiplying both side of Eq. (14) by $Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$ $(\cosh 2\xi-\cos 2\eta)$

and integrating with respect to η from 0 to 2π, and with respect to ξ from 0 to ξ_w.

The constant C_m is determined as follows

$$-\left(V_iF_i(\xi,\eta)-V_0F_0(\xi,\eta)\right)$$
$$Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})\ (\cosh 2\xi-\cos 2\eta)$$
$$=\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} f'(\mu_{m,i},0)$$
$$C_{m,i}Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$
$$Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})\ (\cosh 2\xi-\cos 2\eta)$$
$$-V_i\int_{\eta_{i1}}^{\eta_{i2}}\int_{\xi_{i1}}^{\xi_{i2}} Ce_m(\xi,s_{m,i})\,ce_m(\eta,s_{m,i})$$
$$(\cosh 2\xi-\cos 2\eta)d\xi d\eta$$
$$-V_0\int_{\eta_{01}}^{\eta_{02}}\int_{\xi_{01}}^{\xi_{02}} Ce_m(\xi,s_{m,i})\,ce_m(\eta,s_{m,i})$$
$$(\cosh 2\xi-\cos 2\eta)d\xi d\eta$$
$$=\int_0^{2\pi}\int_0^{\xi_w} f'(\mu_{m,i},0)C_{m,i}Ce_m^2(\xi,s_{m,i})$$
$$ce_m^2(\eta,s_{m,i})(\cosh 2\xi-\cos 2\eta)d\xi d\eta$$

$$C_{m,i}=-\frac{V_i\,G_{m,i}+V_0\,H_{m,i}}{f'(\mu_{m,i},0)} \qquad (15)$$

where

$$G_{m,i}=\frac{\int_{\eta_{i1}}^{\eta_{i2}}\int_{\xi_{i1}}^{\xi_{i2}} Ce_m(\xi,s_{m,i})\,ce_m(\eta,s_{m,i})}{\int_0^{2\pi}\int_0^{\xi_w} Ce_m^2(\xi,s_{m,i})ce_m^2(\eta,s_{m,i})}$$
$$\frac{(\cosh 2\xi-\cos 2\eta)d\xi d\eta}{(\cosh 2\xi-\cos 2\eta)d\xi d\eta} \qquad (16)$$

$$H_{m,i}=\frac{\int_{\eta_{01}}^{\eta_{02}}\int_{\xi_{01}}^{\xi_{02}} Ce_m(\xi,s_{m,i})\,ce_m(\eta,s_{m,i})}{\int_0^{2\pi}\int_0^{\xi_w} Ce_m^2(\xi,s_{m,i})ce_m^2(\eta,s_{m,i})}$$
$$\frac{(\cosh 2\xi-\cos 2\eta)d\xi d\eta}{(\cosh 2\xi-\cos 2\eta)d\xi d\eta} \qquad (17)$$

By multiplying both side of Eq. (14) by $Se_{m+1}(\xi,\bar{s}_{m+1,i})$ $se_{m+1}(\eta,\bar{s}_{m+1,i})$

$(\cosh 2\xi-\cos 2\eta)$ and integrating with respect to η from 0 to 2π, and with respect to ξ from 0 to ξ_w. The constant $S_{m+1,i}$ is determined as follows

$$S_{m+1,i} = -\frac{V_i\ \bar{G}_{m+1,i} + V_0\ \bar{H}_{m+1,i}}{f'(\bar{\mu}_{m+1,i},0)} \qquad (18)$$

where

$$\bar{G}_{m+1,i} = \frac{\int_{\eta_{i1}}^{\eta_{i2}}\int_{\xi_{i1}}^{\xi_{i2}} Se_{m+1}(\xi,\bar{s}_{m+1,i})}{\int_0^{2\pi}\int_0^{\xi_w} Se_{m+1}^2(\xi,\bar{s}_{m+1,i})}$$

$$\frac{se_{m+1}(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta}{se_{m+1}^2(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \qquad (19)$$

$$\bar{H}_{m+1,i} = \frac{\int_{\eta_{01}}^{\eta_{02}}\int_{\xi_{01}}^{\xi_{02}} Se_{m+1}(\xi,\bar{s}_{m+1,i})}{\int_0^{2\pi}\int_0^{\xi_w} Se_{m+1}^2(\xi,\bar{s}_{m+1,i})}$$

$$\frac{se_{m+1}(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta}{se_{m+1}^2(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \qquad (20)$$

Substituting Eq.(15) and Eq.(18) in Eq.(13), the velocity potential can be find as

$$\phi = -\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} \frac{f(\mu_{m,i},z)}{f'(\mu_{m,i},0)}\left(V_i\, G_{m,i} + V_0\, H_{m,i}\right)$$

$$Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$

$$-\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} \frac{f(\bar{\mu}_{m+1,i},z)}{f'(\bar{\mu}_{m+1,i},0)}\left(V_i\ \bar{G}_{m+1,i} + V_0\ \bar{H}_{m+1,i}\right)$$

$$Se_{m+1}(\xi,\bar{s}_{m+1,i})se_{m+1}(\eta,\bar{s}_{m+1,i})$$

$$= \sum_{m=0}^{\infty}\sum_{i=0}^{\infty} \frac{\cosh \mu_{m,i}(z-L)}{\mu_{m,i}\sinh \mu_{m,i}L}\left(V_i\, G_{m,i} + V_0\, H_{m,i}\right)$$

$$Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$

$$+\sum_{m=0}^{\infty}\sum_{i=0}^{\infty} \frac{\cosh \bar{\mu}_{m+1,i}(z-L)}{\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i}L}\left(V_i\ \bar{G}_{m+1,i} + V_0\ \bar{H}_{m+1,i}\right)$$

$$Se_{m+1}(\xi,\bar{s}_{m+1,i})se_{m+1}(\eta,\bar{s}_{m+1,i}) \qquad (21)$$

Therefore, the sound pressure of input becomes

$$P = jk\rho c\phi(\xi,\eta,0)$$

$$= jkZ_w \sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\left(U_i \frac{\cosh \mu_{m,i}L}{\mu_{m,i}\sinh \mu_{m,i}L}\frac{S_w}{S_0}G_{m,i}\right.$$

$$\left. -U_0 \frac{\cosh \mu_{m,i}L}{\mu_{m,i}\sinh \mu_{m,i}L}\frac{S_w}{S_L}H_{m,i}\right)$$

$$Ce_m(\xi,s_{m,i})ce_m(\eta,s_{m,i})$$

$$+ jkZ_w \sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\left(U_i \frac{\cosh \bar{\mu}_{m+1,i}L}{\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i}L}\frac{S_w}{S_0}\bar{G}_{m+1,i}\right.$$

$$\left. -U_0 \frac{\cosh \bar{\mu}_{m+1,i}L}{\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i}L}\frac{S_w}{S_L}\bar{H}_{m+1,i}\right)$$

$$Se_{m+1}(\xi,\bar{s}_{m+1,i})se_{m+1}(\eta,\bar{s}_{m+1,i}) \qquad (22)$$

Similarly, using the relation $P_{out} = jk\rho c\phi(\xi,\eta,L)$ the sound pressure at the output piston can be determined.

The average sound pressure acting on the output can be expressed as

$$\bar{P}_{out} = \frac{1}{S_L}\int_{\eta_{L1}}^{\eta_{L2}}\int_{\xi_{L1}}^{\xi_{L2}} P_{out}\, ds_1\, ds_2$$

$$= jkZ_w \sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\left(U_0 \frac{1}{\mu_{m,i}\sinh \mu_{m,i}L}H_{m,i}^{avr}\right.$$

$$\left. -U_L \frac{\cosh \mu_{m,i}L}{\mu_{m,i}\sinh \mu_{m,i}L}G_{m,i}^{avr}\right)$$

$$+ jkZ_w \sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\left(U_0 \frac{1}{\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i}L}\bar{H}_{m+1,i}^{avr}\right.$$

$$\left. -U_L \frac{\cosh \bar{\mu}_{m+1,i}L}{\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i}L}\bar{G}_{m+1,i}^{avr}\right) \qquad (23)$$

where k is wave-number, $Z_w = \rho c / S_w$, U_0 and U_L are the volume velocity of the input and output section, other symbols are defined by

$$H^{avr}_{m,i} = \frac{q^2}{2}\frac{S_w}{S_0^2}\frac{\left(\int_{\eta_{01}}^{\eta_{02}}\int_{\xi_{01}}^{\xi_{02}} Ce_m(\xi,s_{m,i})\right.}{\int_0^{2\pi}\int_0^{\xi_w} Ce_m^2(\xi,s_{m,i})}$$

$$\frac{\left.ce_m(\eta,s_{m,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta\right)^2}{ce_m^2(\eta,s_{m,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \quad (24)$$

$$\bar{H}^{avr}_{m+1,i} = \frac{q^2}{2}\frac{S_w}{S_0^2}\frac{\left(\int_{\eta_{01}}^{\eta_{02}}\int_{\xi_{01}}^{\xi_{02}} Se_{m+1}(\xi,\bar{s}_{m+1,i})\right.}{\int_0^{2\pi}\int_0^{\xi_w} Se_{m+1}^2(\xi,\bar{s}_{m+1,i})}$$

$$\frac{\left.se_{m+1}(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta\right)^2}{se_{m+1}^2(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \quad (25)$$

$$G^{avr}_{m,i} = \frac{q^2}{2}\frac{S_w}{S_0 S_L}\frac{\left(\int_{\eta_{L1}}^{\eta_{L2}}\int_{\xi_{L1}}^{\xi_{L2}} Ce_m(\xi,s_{m,i})\right.}{\int_0^{2\pi}\int_0^{\xi_w} Ce_m^2(\xi,s_{m,i})}$$

$$\frac{\left.ce_m(\eta,s_{m,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta\right)^2}{ce_m^2(\eta,s_{m,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \quad (26)$$

$$\bar{G}^{avr}_{m+1,i} = \frac{q^2}{2}\frac{S_w}{S_0 S_L}\frac{\left(\int_{\eta_{L1}}^{\eta_{L2}}\int_{\xi_{L1}}^{\xi_{L2}} Se_{m+1}(\xi,\bar{s}_{m+1,i})\right.}{\int_0^{2\pi}\int_0^{\xi_w} Se_{m+1}^2(\xi,\bar{s}_{m+1,i})}$$

$$\frac{\left.se_{m+1}(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta\right)^2}{se_{m+1}^2(\eta,\bar{s}_{m+1,i})(\cosh 2\xi - \cos 2\eta)d\xi d\eta} \quad (27)$$

$$\mu_{m,i} = \frac{1}{a_w}\sqrt{\lambda_{m,i}^2 - \left(ka_w\right)^2} \quad (28)$$

$$\bar{\mu}_{m+1,i} = \frac{1}{a_w}\sqrt{\bar{\lambda}_{m+1,i}^2 - \left(ka_w\right)^2} \quad (29)$$

3. RESULTS AND DISCUSSIONS

Hereafter, in order to obtain a great soundproofing effect, we will investigate the characteristic of the higher order mode that propagated inside the unit. Resonance of the higher order mode wave will occur when the denominator of Eq. (23) become zero, namely at the frequencies of

$$\mu_{m,i}\sinh \mu_{m,i} L = 0$$

$$\therefore \quad f_{m,i} = \frac{c}{2\pi a_w}\sqrt{\lambda_{m,i}^2 + \left(n\pi a_w / L\right)^2}$$

$$(n = 0, 1, 2, ...) \quad (30)$$

$$\bar{\mu}_{m+1,i}\sinh \bar{\mu}_{m+1,i} L = 0$$

$$\therefore \quad \bar{f}_{m,i} = \frac{c}{2\pi a_w}\sqrt{\bar{\lambda}_{m+1,i} + \left(n\pi a_w / L\right)^2}$$

$$(n = 0, 1, 2, ...) \quad (31)$$

The constants $\lambda_{m,i}$ and $\bar{\lambda}_{m+1,i}$ corresponding to eccentricity of the elliptical ventilation unit are given in [3]. Generation mechanism of the higher mode can be understood according to the calculation example of even and odd modes within 4kHz shown in **Fig.3** in the case of elliptical having a eccentricity of 0.74, L = 12cm and the major axis of 7.8cm. **Fig.4** shows the experimental result. Agreement observed between the measurement and our predicted resonance frequencies is acceptable. Note that, the symbol C in **Fig. 4** related to the Insert-Loss, it be obtain by $C = U_0 / \bar{P}_{out}$ while U_L=0.

4. CONCLUSIONS

Characteristic of sound propagation in an elliptical soundproofing unit having an input and output has been presented by solving the wave equation considering the higher-order mode effect. To prove the theory, experiments were carried out and agreement is obtained. Eq.(23) enable account the insertion-loss or the selection of size and placement of input and output openings in such a way that would minimize the effect of higher-order mode waves.

5. REFERENCES

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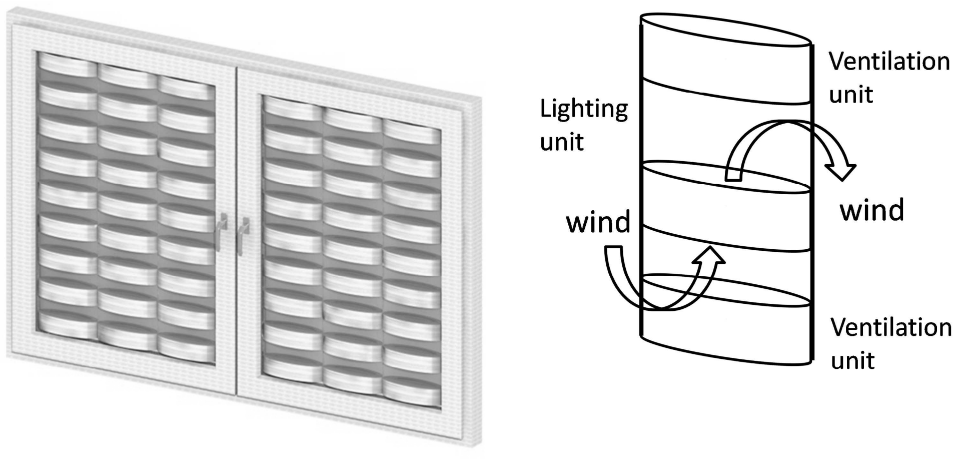

Figure1. Basic structure of the windows

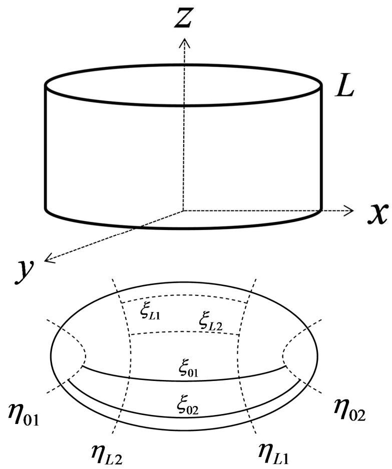

Figure2. Model of calculation

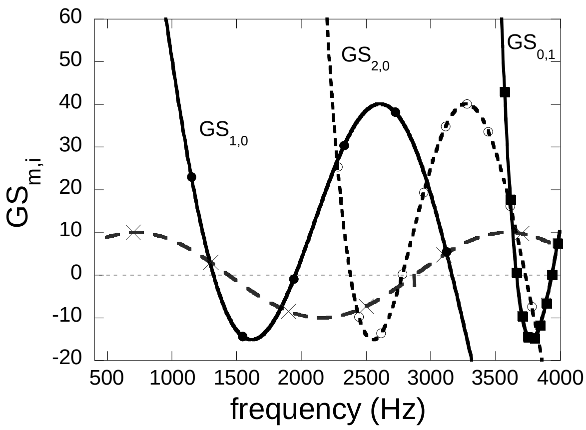

Figure3. Computed result based on Eq.(6)

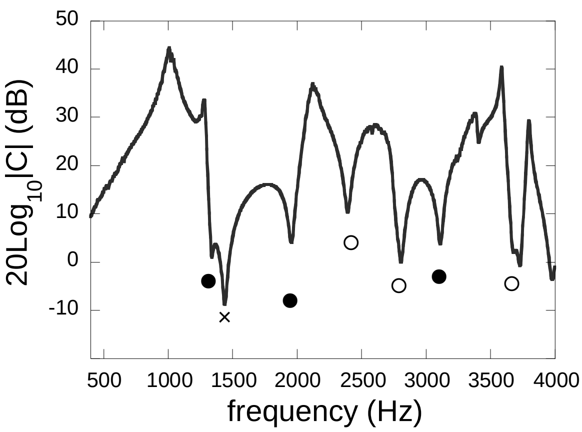

Figure4. Experimental result